The Effect of Thermal-mechanical History on the Strain Hardening of Metals

By J. E. Dorn,* Member AIME, A. Goldberg† and T. E. Tietz†

(Philadelphia Meeting October 1948)

Introduction

The concept that the flow stress for plastic deformation of metals in the work hardening range is a function of the instantaneous values of the strain, strain rate and test temperature was promulgated by Ludwick[1] many years ago. This thought has been more or less a guiding principle throughout the immediate period of development and growth of mechanical metallurgy. It has been tacitly assumed or knowingly applied to many investigations on the plastic behavior of metals. Spurred by the theoretical interest and practical importance of the plastic behavior of metals, many investigators have attempted to determine empirical, semi-empirical, and theoretical equations[2-18] relating the flow stress with the strain, strain rate, and temperature. Although the applications of such equations have been extensive, the experimental verification of the existence of a mechanical equation of state is weak. Furthermore, some of the available evidence on plastic flow appears to suggest that no simple functional relation between stress, strain-rate, strain and temperature can exist.

The mechanical equation of state demands that

$$\sigma = \sigma(\epsilon, \dot{\epsilon}, T) \qquad [1]$$

Where σ = true stress = load/instantaneous area

ϵ = true strain

$$= \ln \frac{\text{Instantaneous gauge length}}{\text{initial gauge length}}$$

$\dot{\epsilon}$ = true strain rate

T = temperature

This formulation implies that $d\sigma$ is an exact differential. Therefore the flow stress (σ) is a function of the instantaneous values of the strain (ϵ) the strain rate ($\dot{\epsilon}$) and the temperature (T), independent of the previous mechanical or thermal history. Having reached a certain strain, strain rate, and temperature, by any path whatsoever, a definite and fixed value of the flow stress should be obtained.

One check on the validity of the mechanical equation of state is easily obtained. Consider the following three tests on the stress-strain curve at constant temperature:

A. Strain at rate $\dot{\epsilon}_1$

B. Strain at rate $\dot{\epsilon}_2$

C. Strain at rate $\dot{\epsilon}_1$ to ϵ_1 and continue test at $\dot{\epsilon}_2$.

A graphical representation of possible results of such tests are shown in Fig 1. If, for strains greater than ϵ_1, test (C) data coincide exactly with the data from test (B), as shown in Fig 1a, a limited verification of the mechanical equation of state is obtained for the range of strain rates from $\dot{\epsilon}_1$ to $\dot{\epsilon}_2$. But if the data from test (C) differ from those of (B) beyond strain ϵ_1 where the strain rate is changed from $\dot{\epsilon}_1$ to $\dot{\epsilon}_2$, the mechanical equation of state is not

---

valid and the plastic properties depend upon the past history of strain rates.

Another simple check on the possible validity of the mechanical equation of

tempered alloy steel between 70 and 900°F (for small strains and a short time interval at 900°F) appears to confirm the existence of the mechanical equation of state over the
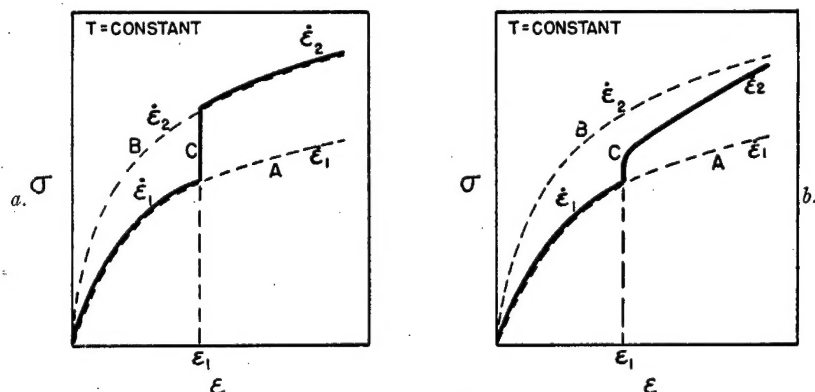

FIG 1—HYPOTHETICAL RESULTS INDICATING VERIFICATION OR FAILURE OF THE MECHANICAL EQUATION OF STATE WHEN THE STRAIN RATE IS CHANGED FOR CONSTANT TEST TEMPERATURE.
a. Verification of a mechanical equation of state.
b. Failure of a mechanical equation of state.

state can be obtained from stress-strain curves at two temperatures for constant strain rates from the following tests:

A. Strain at T_1

B. Strain at T_2

C. Strain at T_2 to ϵ_1 and continue the test at T_1.

Possible results of such tests are shown in Fig 2. If curve C agrees with A beyond ϵ_1, as shown in Fig 2a, partial verification of the mechanical equation of state is obtained over the range of temperatures from T_1 to T_2. But if the data from test (3) do not coincide exactly with those from test (1) beyond strain ϵ_1 where the temperature was changed from T_1 to T_2, as shown in Fig 2b, the mechanical equation of state is invalid and the plastic properties are dependent upon past thermal-mechanical history.

Hollomon[8] performed two tests of the kind outlined in Fig 2 and concluded that, "the stress required for plastic flow depends only on the instantaneous values of the strain rate, temperature and strain, and not upon the past history of these variables." His test on a quenched and

test variables. The broad application of these results for formulating an analysis of creep, however, is not valid and will be discussed in a future report.

The second test series conducted at -105 and $+70°F$ on an SAE steel, which is reproduced in Fig 3, does not, however, appear to present conclusive evidence in favor of the mechanical equation of state. The sampling was poor since test bar (C) exhibited higher stresses than (A) over the range of strains to about 0.38. When, however, test (C) was continued at $-105°F$ the flow stress fell below that for (B) suggesting that, with careful sampling, the data might have yielded a curve analogous to that illustrated in Fig 2b. It is, therefore, reasonable to suspect that these data are inconclusive and that they may actually suggest the failure of the mechanical equation of state over low temperature ranges.

Kauzmann[9] and also Dushman et al[7] have shown that when Eyring's reaction rate theory is applied to the secondary creep rate of metals, the theoretical size of the unit participating in plastic flow, increases

with increasing temperature. Disregarding, for the present, the possible inadequacy of Kauzmann's theory, the reported effect of temperature on the size of the flow unit

dislocations or Kauzmann's analysis for creep are not convincing because reasonable doubt persists concerning the accuracy of the mechanisms which were postulated for
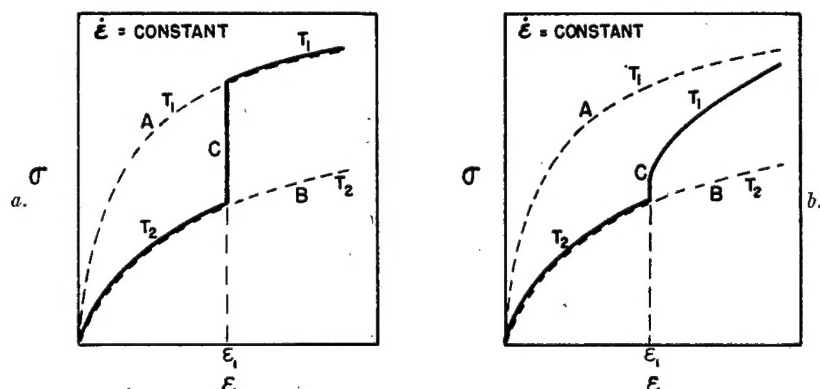

FIG 2—HYPOTHETICAL RESULTS INDICATING VERIFICATION OR FAILURE OF THE MECHANICAL EQUATION OF STATE WHEN THE TEMPERATURE IS CHANGED FOR CONSTANT STRAIN RATE.

a. Verification of a mechanical equation of state.
b. Failure of a mechanical equation of state.

suggests the failure of the mechanical equation of state. If large flow units are generated at elevated temperatures, such units persist for the first stages of straining at lower temperatures, causing the flow stress at the lower temperatures to be less after prestraining at elevated temperatures than they would have been if the entire test had been conducted at the lower temperature. The experimental data reported later verifies this prediction.

Applying the dislocation theory to aluminum single crystals at subatmospheric temperatures, Taylor[15] concluded that the opaque surfaces, postulated to account for work hardening, were closer together at the lower temperatures. This statement is somewhat equivalent to that given by Kauzmann concerning the size of the flow unit for creep. Therefore, the pattern of dislocations, stopped at opaque surfaces, due to prestraining at atmospheric temperature, would be on a larger scale than if the test had been made at lower temperatures. Consequently a temperature history dependence of the plastic properties would be expected.

Deductions based on Taylor's theory of

deformation, strain hardening, and plastic flow in these approaches. But when other data are compounded on these deductions, the evidence favoring history dependence of plastic properties begins to assume substantial proportions. It is necessary merely to mention here the well known effects of decreasing temperatures and increasing strain rates on the distances between slip bands in single metal crystals. Straining a single crystal at a given temperature or strain rate produces greater distances between the slip bands than would be obtained by straining the same amount at lower temperatures or higher strain rates. Undoubtedly the internal structure of the metal has been modified in a manner correlateable with the observed external spacing of the slip bands. Prestraining at one temperature or strain rate, therefore, should yield a different stress-strain relationship when the conditions are changed than would have been obtained if the entire straining had been conducted exclusively under the second set of conditions.

It is the purpose of this investigation to ascertain from tension tests at various temperatures and changes of temperature

whether a mechanical equation of state exists or whether the past thermal-mechanical history affects the plastic properties of metals.

gauge yielded strains accurate to ±0.001. All tests were conducted at a true strain rate of approximately 0.0011 per sec.

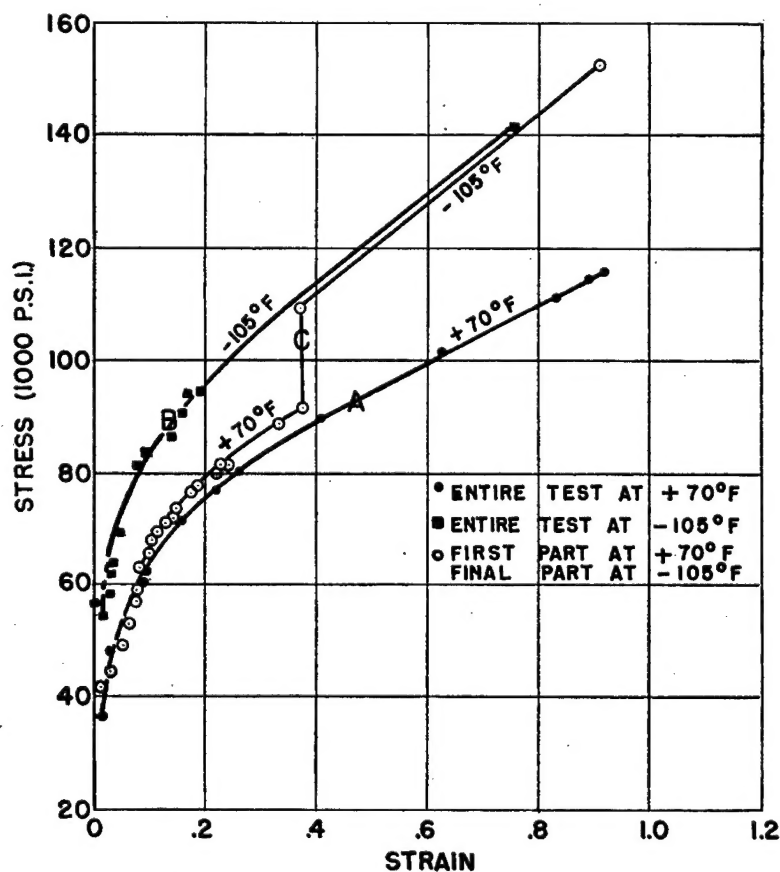The load was measured by means of a

FIG 3—EFFECT OF PRE-STRAINING AT 70°F ON THE TRUE STRESS-TRUE STRAIN CURVE AT −105°F
(*Data by Hollomon*).

MATERIALS AND TESTING TECHNIQUES

Commercially pure aluminum (2S-O annealed at 1000°F for 20 min.) was the principal test material. In order to determine whether the observed effects were peculiar to 2S-O, additional tests were conducted on high purity (99.98 pct) aluminum, brass, copper and stainless steel.

A rack and pinion strain gauge having a 2-in. ga. length was used for measuring extensions. Calibration showed that this

proving ring containing a dial gauge, sensitive to ±½ lb. Good reproducibility was obtained on duplicate tensile tests.

The test specimen design is illustrated in Fig 4. All specimens were machined with their longitudinal axes in the rolling direction. The machined edges were carefully polished with 3/0 emery paper. The extra long ½ in. parallel section at either end of the 2-inch ga. length was introduced in order to assure uniform tensile straining

over the gauge section. The width and thickness of the specimens were gauged with a 0.0001-in. hand micrometer.

The tests at 292°K were conducted in

EXPERIMENTAL RESULTS

The experimental results are given in Fig 6 to 17. All data are recorded in the form of true stress (σ = load per unit
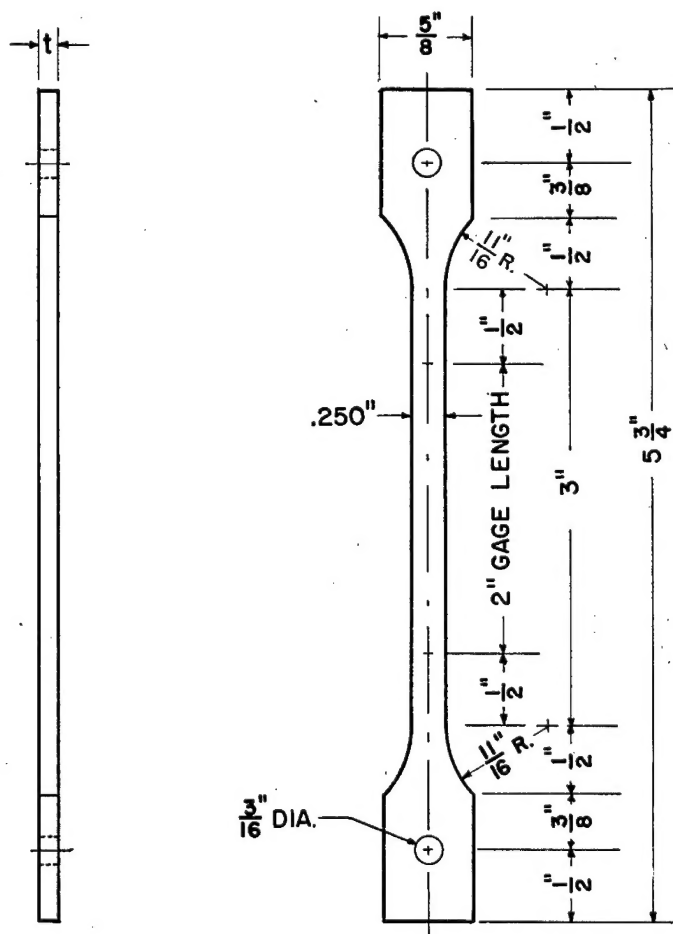

FIG 4—TENSILE SPECIMEN.

the atmosphere but the 260, 194 and 78°K tests were carried out with the specimen mounted in a special dewar container in direct contact with ice brine, CO_2-acetone, and liquid nitrogen respectively. Thermocouples on the specimens showed that the specimen temperatures were within ±1° of the reported temperatures over the 2-in. ga. section throughout each test. A typical test set up is shown in Fig 5.

instantaneous area) vs. true strains (ϵ = natural logarithm of the instantaneous length divided by the initial gauge length). All strains are recorded as plastic strains which were obtained by subtracting the elastic strains from the total true strains.

In Fig 6 is shown a typical set of stress-strain curves carried out at 4 temperatures; the lowest curve (292°K) was obtained at room temperature whereas the highest

curve ($78°K$) was obtained at liquid nitrogen temperatures. Duplicate tests reproduced the stress to within about ± 50 psi. The point N refers to the strain

rupted when a change was made from one temperature to another, it was deemed advisable to check the effect of such interruptions on the stress-strain curve. The
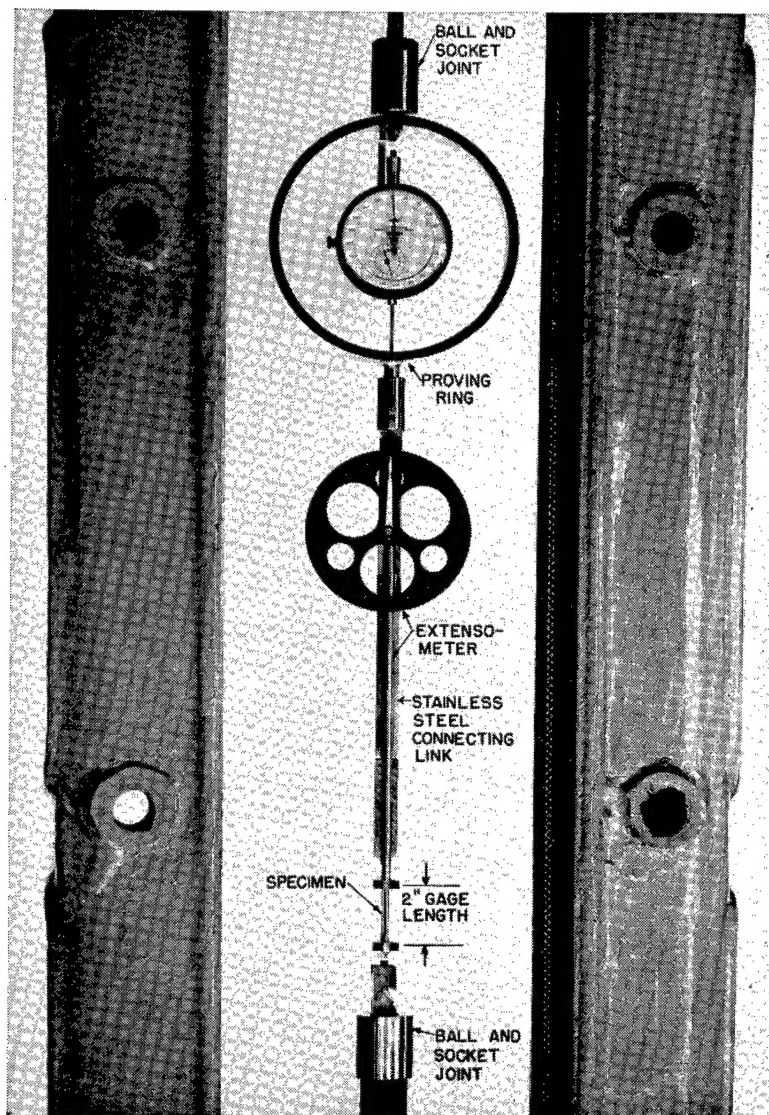

FIG 5—TYPICAL TEST SET-UP.

at which necking begins, or at which the maximum load was obtained.

Since checking the mechanical equation of state demanded that the tests be inter-

data recorded in Fig 7 reveal that after unloading and reloading at a strain of about 0.15 no discontinuity in the stress-strain curve was observable. The inter-

rupted stress-strain curve of Fig 7 agrees very well with the uninterrupted test curve for 78°K. The effect of interruptions at room temperature, shown in Fig 8,

state were valid the curves, after changing the temperature to 78°K, should have followed the highest broken curve which was reproduced from the data of Fig 6 for
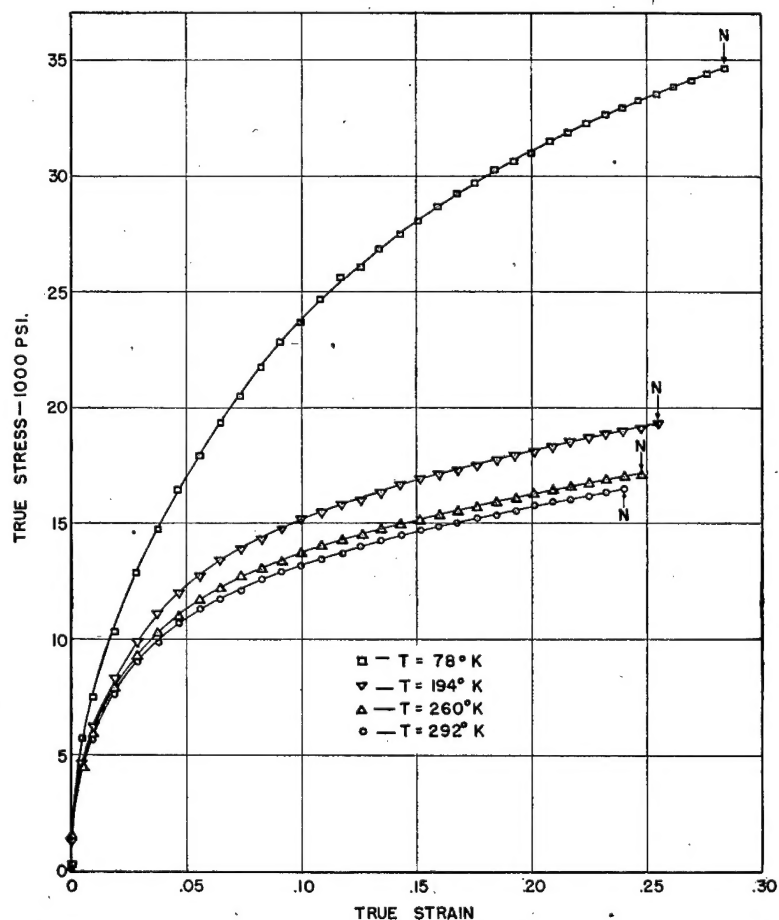

FIG 6—EFFECT OF TEST TEMPERATURE ON TRUE STRESS-TRUE STRAIN CURVE FOR 2S-O.

is also negligible proving that within the range of temperatures used in this study there is no significant effect of interruptions on the stress-strain curves.

Fig 9 to 11 illustrate the effect of prestraining at 292, 260, and 194°K on the stress-strain curves at 78°K. The strain at which each specimen was unloaded and the temperature changed is indicated by an appropriate symbol at approximately zero stress. If the mechanical equation of

the continuous test at 78°K. The difference between the broken curve and those obtained after prestraining at higher temperatures increases with increasing prestrain. A regular increase in the total strain to necking with increasing prestrain at the higher temperature is also apparent.

In Fig 12 and 13 are shown the effects of prestraining at 78°K on the stress-strain curves at 292 and 194°K. Here again

the mechanical equation of state is found to fail.

Because the failure of the mechanical equation of state for 2S-O might have in the test series because it was suspected that it would behave anomolously due to the reported formation of ferrite during deformation. These suspicions were con-
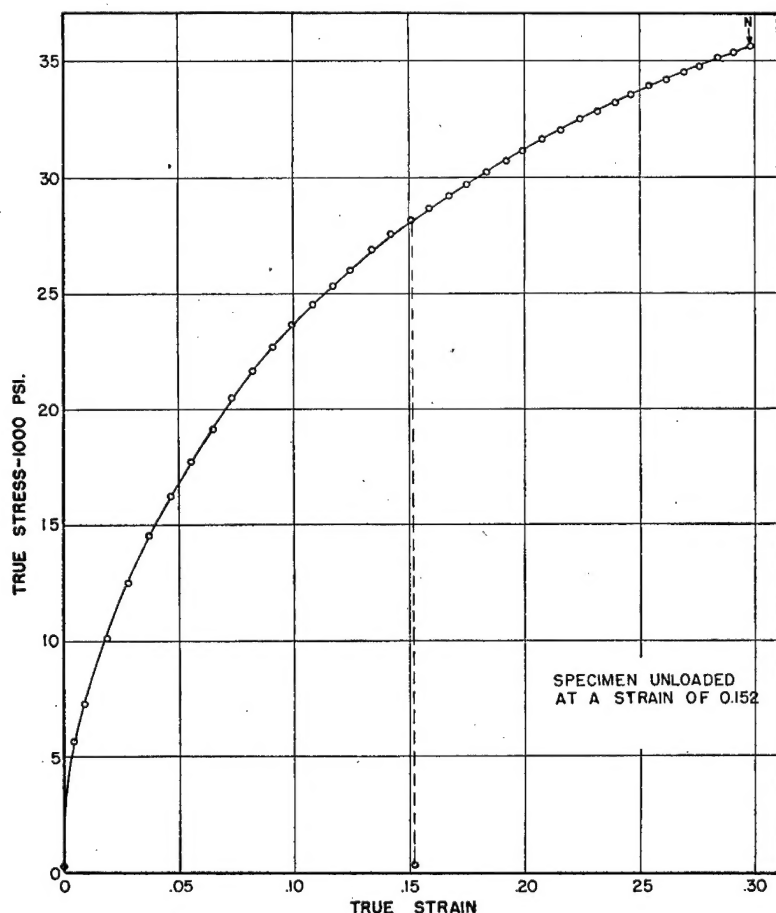

FIG 7—EFFECT OF UNLOADING SPECIMEN DURING TEST ON TRUE STRESS-TRUE STRAIN CURVE AT 78°K FOR 2S-O.

been due to peculiarities of this alloy, tests were also conducted on pure aluminum, copper, brass and stainless steel (18-8) as recorded in Fig 14 to 17 inclusively. These data strongly suggest that the failure of the mechanical equation of state is general for metals that crystallize with the face centered cubic structure and that similar failure might be expected for all other systems. Stainless steel was included

firmed by the data shown in Fig 17 which exhibit several peculiarities that are worthy of additional study.

DISCUSSION OF RESULTS

The data presented in Fig 9 are analogous to those shown in Fig 2a and therefore prove that the mechanical equation of state is invalid for 2S-O alloy between liquid nitrogen and atmospheric tempera-

ture. The deviations from the dictates of the mechanical equation in the extreme case amount to as much as 35 pct. Reflection concerning the possibilities of sec-

discontinuous plastic flow in aluminum* reported by McReynolds[20] suggests that 2S-O alloy might be susceptible to strain aging. For this reason, tests were run at
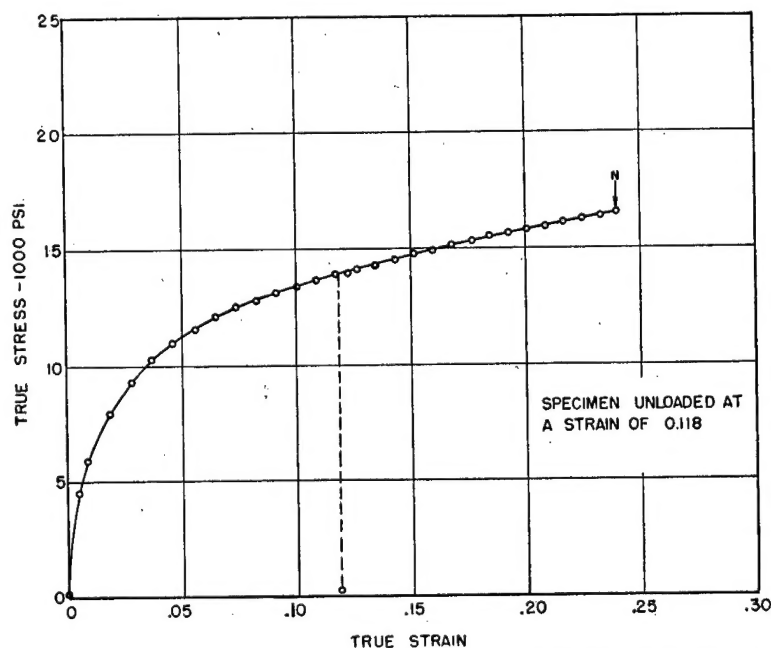

FIG 8—EFFECT OF UNLOADING SPECIMEN DURING TEST ON TRUE STRESS-TRUE STRAIN CURVE AT 292°K FOR 2S-O.

ondary factors entering the analysis to promote the failure of the mechanical equation of state suggested that more extensive studies were required:

1. *Recovery:* Obviously the mechanical equation of state is invalid where crystal recovery is possible. Calvet[19] has shown, however, that as the impurities approach the concentrations in commercially pure aluminum, recovery at atmospheric temperature becomes excessively slow. Nevertheless, it appeared to be advisable to conduct tests at lower temperatures in order to reduce further possible complicating effects of recovery.*

2. *Strain Aging:* The yield stress and

lower temperatures and additional tests were made on high purity aluminum.

3. *Mechanisms of Deformation:* A failure of the mechanical equation of state might be expected for ranges of temperature over which changes in mechanisms of slip occur. Although single crystals of aluminum are thought to exhibit slip only on the {111} planes in [101] directions over the ranges of temperatures which were studied,[21] it seemed that additional tests over lower, narrower ranges of temperature were desirable.

The data recorded in Fig 9, 10, and 11 reveal that the failure of the mechanical equation of state is equally valid though less pronounced for narrower ranges of temperature. Furthermore, the test results

---

* Long term recovery tests on 2S-O are now in progress.

* It is interesting to observe that these effects were found to diminish with decreasing temperatures in this study.

for the high purity aluminum alloy given in Fig 14 indicate that the observations are probably not attributable to strain aging effects.

Now the failure of the mechanical equation of state presents a fundamental problem of serious import to all analyses on plastic deformation of metals. Whereas
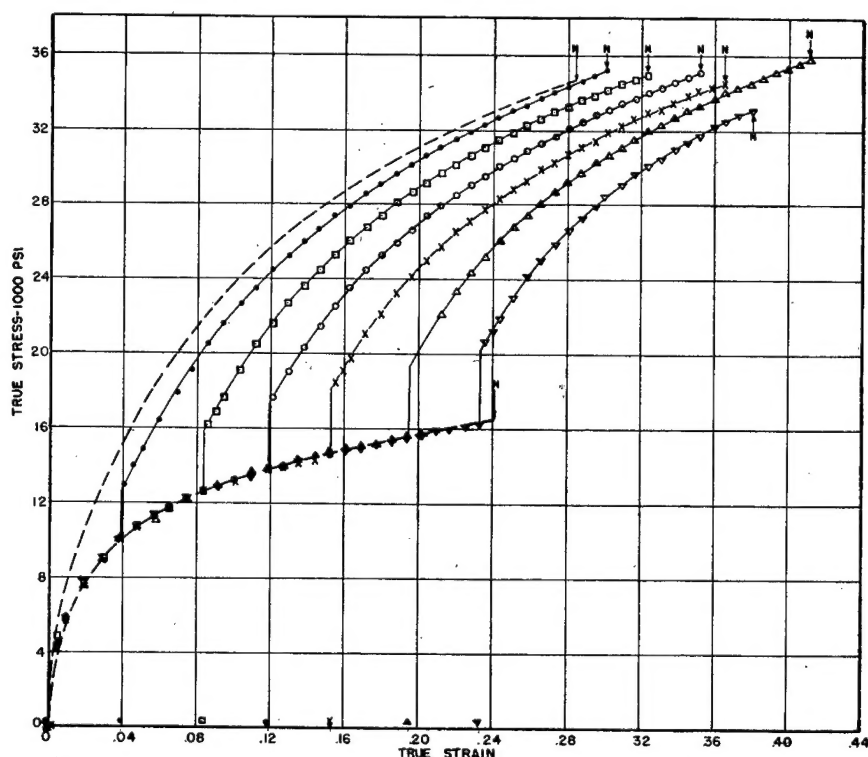

FIG 9—EFFECT OF PRE-STRAINING AT 292°K ON $\sigma - \epsilon$ CURVE AT 78°K FOR 2S-O.

In order to determine whether the failure of the mechanical equation of state is unique for aluminum the series of tests recorded in Fig 15 to 17 was conducted on copper, brass, and 18-8 stainless steel. The data obtained on all of these materials confirmed the failure of the mechanical equation of state. This evidence appears to suggest that the assumption of a mechanical equation of state will have to be abandoned. The flow stress for plastic deformation of metals is not only a function of the instantaneous values of strain, strain rate and temperature, but also depends, in a systematic way, on the previous thermal-mechanical history of deformation.

the existence of the function $\sigma = \sigma(\epsilon, \dot{\epsilon}, T)$ would have permitted the work hardened state to be described in terms of the strain, ϵ, independent of the strain rates and temperatures which were employed to achieve that strain, the history dependence of plastic flow no longer permits the strain to be used to identify the work hardened state. For example, reference to Fig 9 illustrates that for a strain of 0.16 the flow stress at liquid nitrogen temperature and a constant strain rate of $\dot{\epsilon} = 0.0011$ per sec may be 19,300, 23,200, 25,800, 27,800 or 28,800 psi depending upon the previous strain history at atmospheric temperature. A strain of 0.10 at liquid

nitrogen temperatures causes a greater degree of strain hardening and therefore more pronounced structural changes* than the same strain would produce at atmos-

which was obtained for strain at $\epsilon = 0.056$ at liquid nitrogen temperature. Repeating this analysis for the other pre-strains suggests that the strain equivalence is that
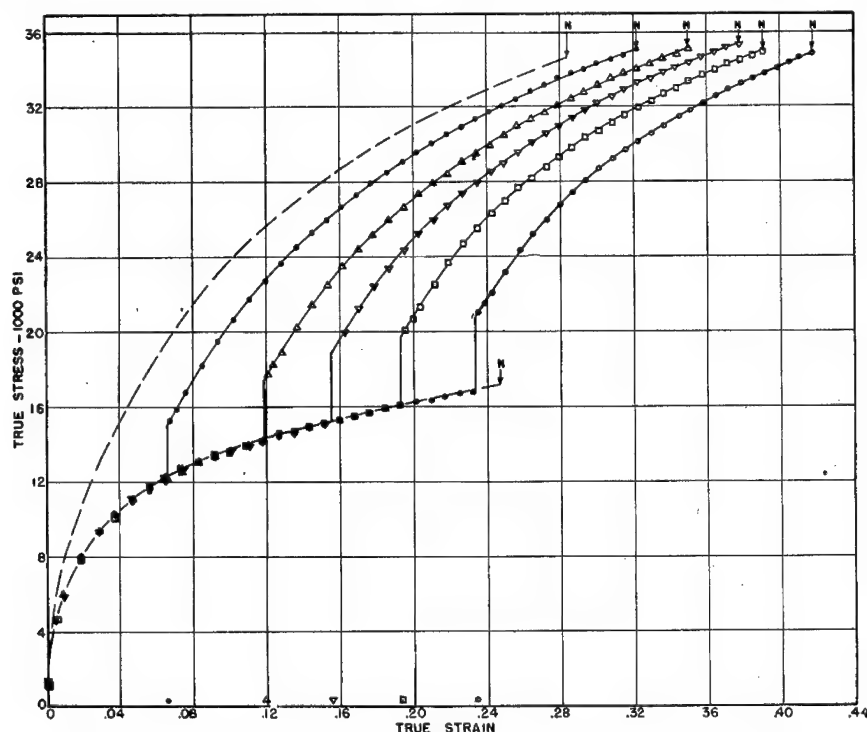

FIG 10—EFFECT OF PRE-STRAINING AT 292°K ON $\sigma - \epsilon$ CURVE AT 78°K FOR 2S-O.

pheric temperatures.† The new parameter which may replace the strain for evaluating plastic properties of metals appeared at first to be some history modified strain. The simplest possibility is that the history modified strain is an equivalent strain; the prospects for the success of this concept appeared to be bright when first reviewed.

Consider again the data of Fig 9. After prestraining $\epsilon = 0.151$ at atmospheric temperature the new flow stress at liquid nitrogen temperature is equal to that

shown in Fig 18 for prestraining at 292°K. If this thought be correct, the data of Fig 9 should not have been plotted in terms of the actual strains at atmospheric temperature but rather in terms of the equivalent strains at liquid nitrogen temperature. Such a plot is shown in Fig 19. Ordinarily such agreement as is obtained between the various tests recorded in Fig 19 would be adequate to substantiate, if not the validity, at least the practical utility of the history modified equivalent strain.

Frequently history dependent phenomena have the perverse habit of yielding different results upon reversals of the path. For this reason the series of tests recorded in Fig 12 and 13 were obtained by pre-

* Preliminary tests on structural changes have been initiated which reveal that greater fragmentation occurs when straining at liquid nitrogen temperature than is obtained by the same strain at room temperature.

† A similar influence of strain rate at constant temperature has been observed.

straining at liquid nitrogen temperature and then concluding the test at a higher temperature. Here again the data demonstrate that the flow stress is dependent upon the past thermal-mechanical history.

Equivalent strains for prestraining at liquid nitrogen temperature and con-

curve obtained by straining exclusively at 78°K. Therefore no true equivalence of work hardened states exists when pure Al is strained at two different temperatures.

The data contained in this report suggest reasons for some previously unexplained observations such as the effect of lowered
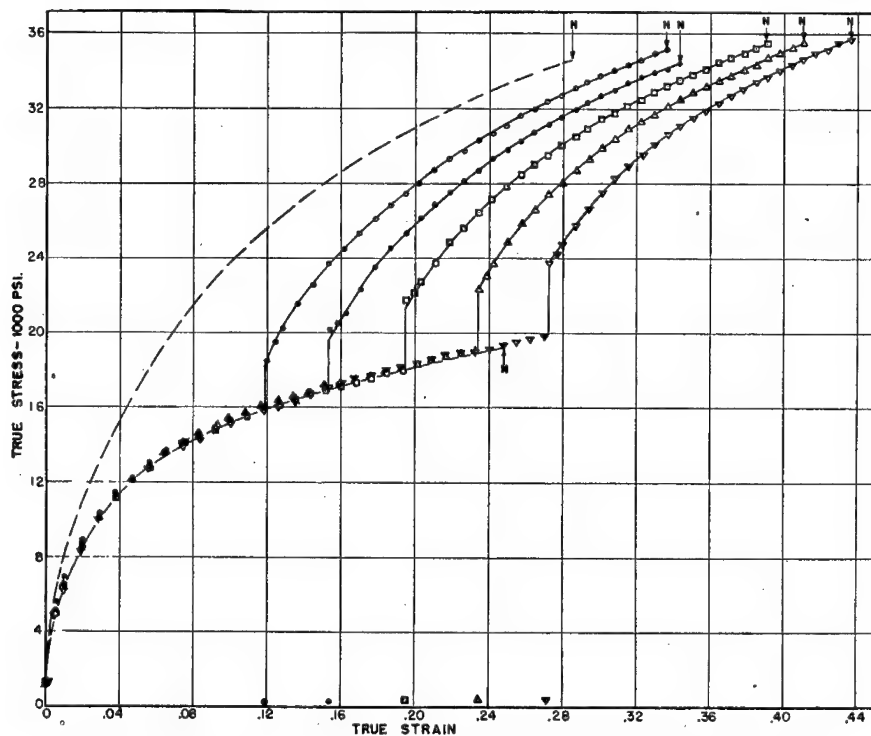

FIG 11—EFFECT OF PRE-STRAINING AT 194°K ON $\sigma - \epsilon$ CURVE AT 78°K FOR 2S-O.

tinuing the tests at atmospheric temperature are shown in the upper curve of Fig 18. Although the equivalent strains obtained by the two alternate paths agree well up to strains of 0.04, they begin to diverge for larger strains. This failure to agree over the entire range of strains proves that the concept of equivalent strains is untenable. The data of Fig 14 also present conclusive evidence against the concept of equivalence of strain. If the stress-strain curve at 78°K, after prestraining 292°K, is shifted to the left, it will coincide at only one point with the stress-strain

temperatures of cold working on reducing the recrystallization temperature. When it is realized that the amount of work hardening is greater for the same strain at a lower temperature, the effect of the temperature of cold work on the recrystallization temperature becomes evident. But, in general, the data reported here uncover more problems than they solve. The most important concerns finding an appropriate parameter for identifying the work hardened state. The present evidence proves that no simple history modified strain parameter exists.

The reasons for the history sensitivity of plastic deformation are not yet clear. Perhaps the phenomenon is associated with relaxations that can occur at low temperatures during plastic deformation. On the basis of this concept less relaxation would be activated at the lower temperatures a simple analysis for plastic flow of metals based on Eyring's reaction rate theory. When simplified, Kauzmann's equation for plastic flow becomes

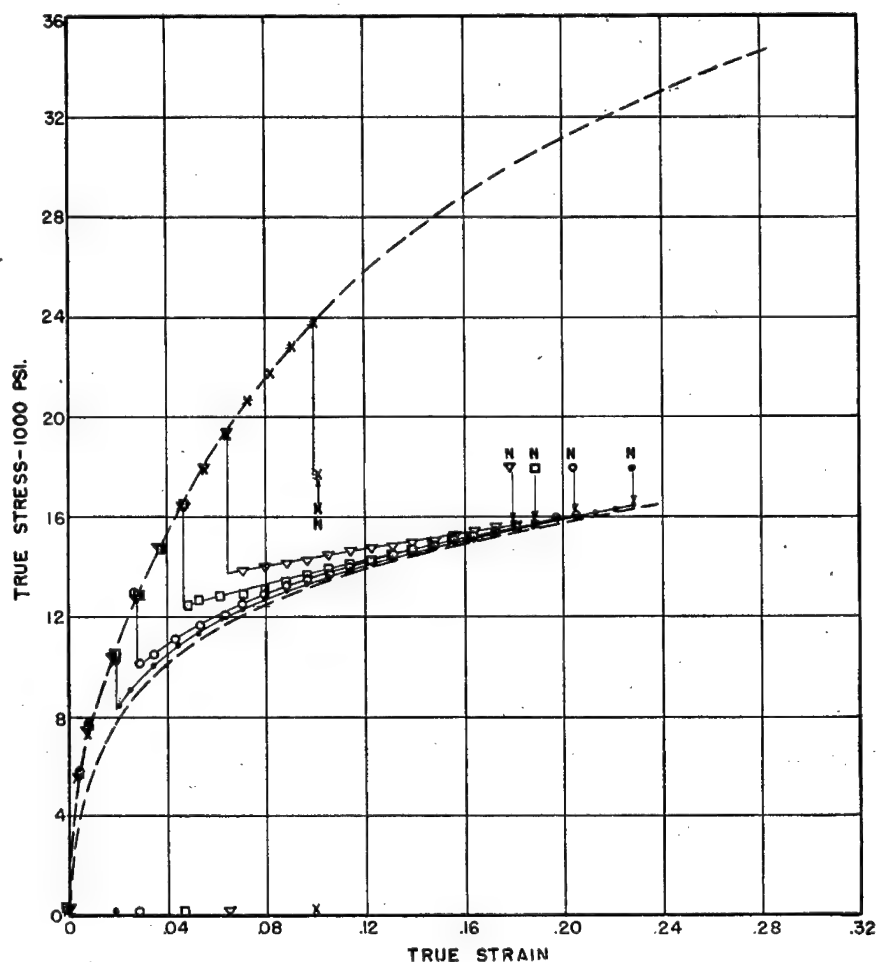$$\dot{\epsilon} = A T e^{\frac{-\Delta H^*}{RT}} e^{B\sigma} \qquad [2]$$

FIG 12—EFFECT OF PRE-STRAINING AT 78°K ON TRUE STRESS-TRUE STRAIN CURVE AT 292°K FOR 2S-O.

and greater amounts of work hardening would result for the same strains. By the same token, higher strain rates would permit fewer relaxations over the same strain and likewise give higher amounts of work hardening.

Several years ago Kauzmann[9] presented

Where

$\dot{\epsilon}$ = strain rate
σ = applied stress
T = absolute temperature
R = gas constant
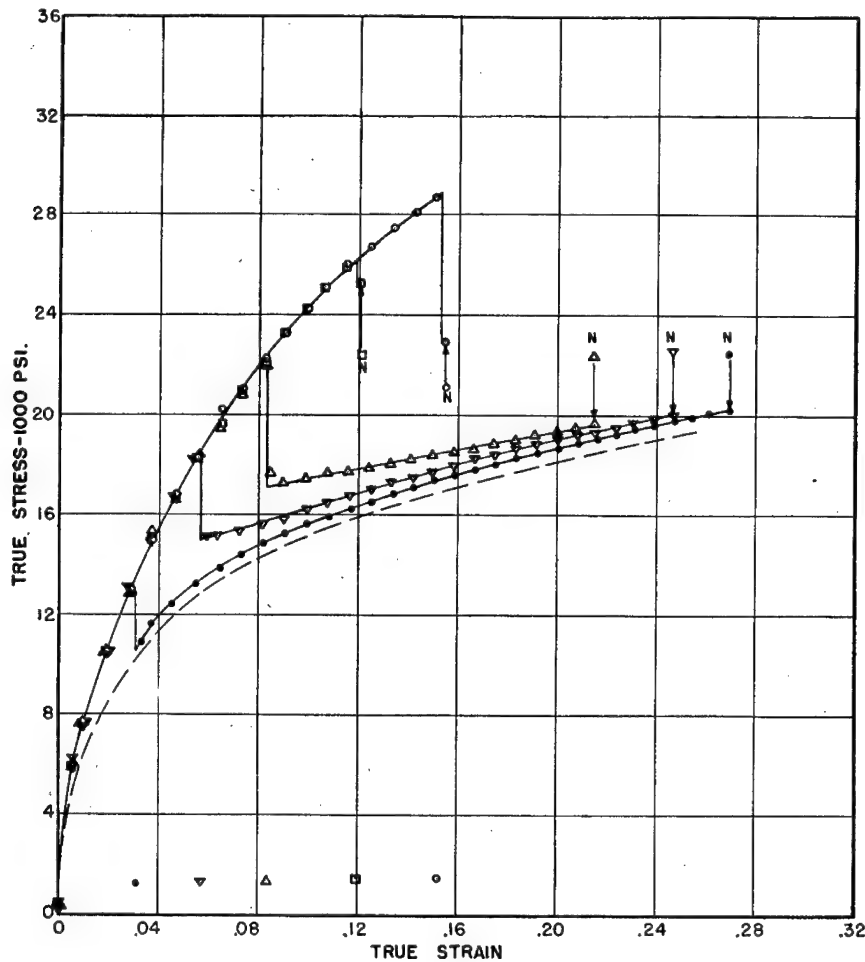ΔH^* = activation energy
A,B = parameters

FIG 13—EFFECT OF PRE-STRAINING AT 78°K ON TRUE STRESS-TRUE STRAIN CURVE AT 194°K FOR 2S-O.

In Kauzmann's formulation

$$A = \frac{2\lambda k}{3Lh} e^{\frac{\Delta S^*}{R}}$$ [3]

$$B = \frac{alq}{kT}$$ [4]

Where

k = Boltzman's constant

h = Plank's constant

ΔS^* = entropy of activation

a = area of flow unit

l = distance the flow unit must be displaced for activation

λ = total shear displacement of a flow unit per activation

L = normal distance between flow units

q = stress concentration factor

Although existing knowledge on the detailed mechanics of plastic flow in metals is not yet adequate to either test or question the validity of Eq 3 and 4, numerous data agree fairly well with the relationships suggested by Eq 2 over restricted ranges of temperature and strain rates.

Eq 2 was proposed principally for analyses of secondary creep rates. Under
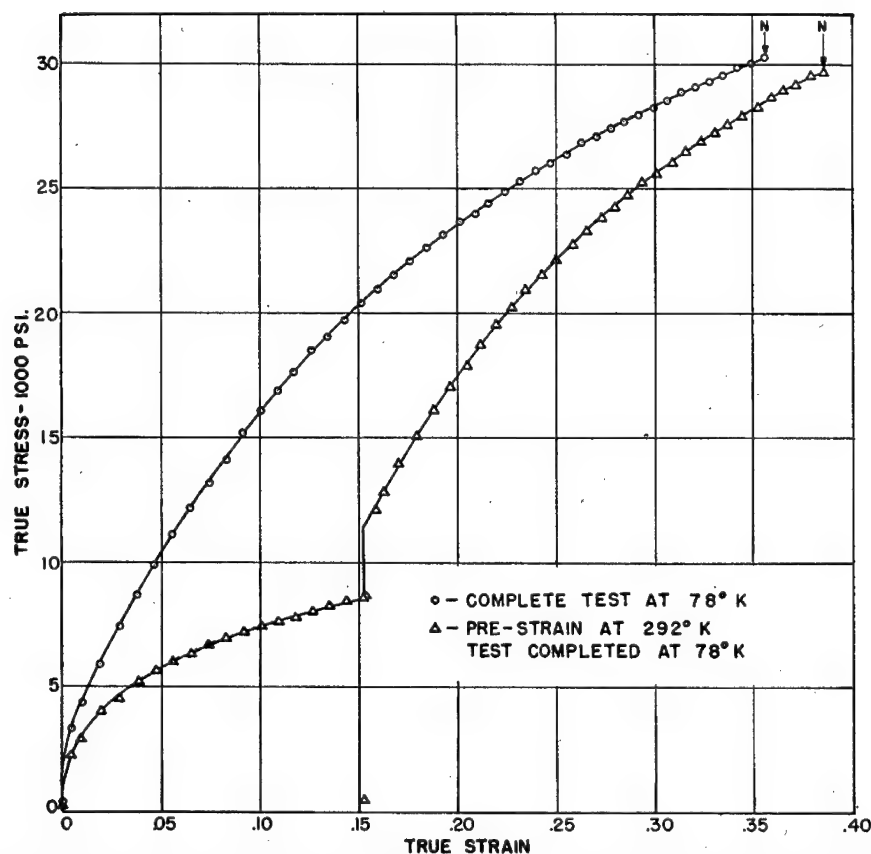
FIG 14—EFFECT OF PRE-STRAINING AT 292°K ON TRUE STRESS-TRUE STRAIN CURVE AT 78°K FOR PURE ALUMINUM (99.98 PCT).

these conditions, the rate of strain hardening is believed to be equal to the rate of softening by recovery. Kauzmann, in effect, assumed that A and ΔH^* would have the same values for all conditions of strain and temperature where secondary creep rates were obtained. The analysis, however, should be applicable to various work hardened states if A, ΔH^* and perhaps B are taken to be functions of strain hardening.

The data reported here, however, uncover a fundamental difficulty in applying Kauzmann's analysis in the strain hardening range of plastic deformation. The parameters A, B and ΔH^* are functions of the work hardened state, and therefore could have been evaluated as functions of strain if the work hardened state were a unique single valued function of the strain. But the work hardened state depends upon the past thermal-mechanical history, and therefore, it is not possible at present to determine A, B or ΔH^* as functions of the strain.

It should be emphasized that, whereas the experimental results reported here disprove the validity of all mechanical equations of state where the flow stress is assumed to be a function of the strain, strain-rate and temperature, they do not disqualify Kauzmann's formulation. This is so because Kauzmann's analysis does not contain the strain as an explicit vari-

able. Rather it contains the parameters *A*, *B* and ΔH^* which now should not be related to the strain but rather to equivalent work hardened states. The major

that major effort be bent toward finding a history modified equivalent strain, the prognosis for success in this direction appears to be negative. If upon cold work-
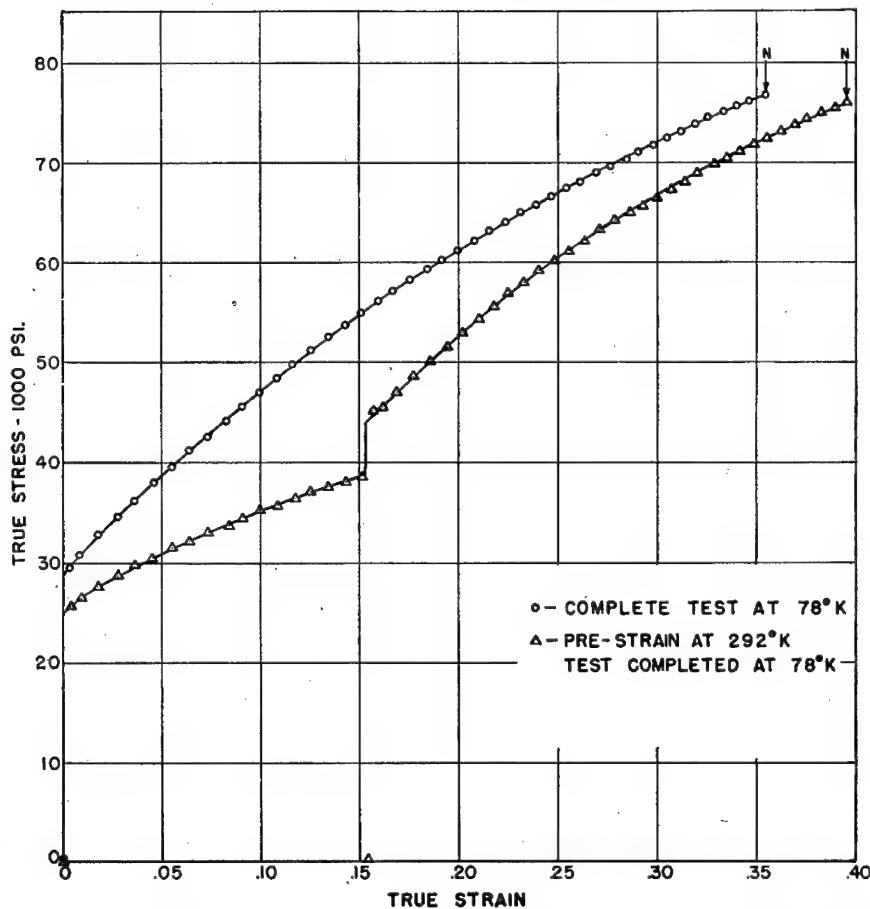

FIG 15—EFFECT OF PRE-STRAINING AT 292°K ON TRUE STRESS-TRUE STRAIN CURVE AT 78°K FOR COPPER.

question then concerns the problem of the existence and possible methods of identifying (or parameterizing) equivalent work hardened states. Until this can be done it will be impossible to correlate intelligently stress-strain data at different strain rates or at different temperatures.

Although the simplicity of analysis that would be achieved if equivalent work hardened states can be identified suggests

ing only one internal structural feature of the metal is modified, then equivalent work hardened states must exist. But if, as appears more likely, work hardening is due to the production of two or more structural changes, equivalent work hardened states (in terms of the meaning being used here) are unlikely.

Consider that work hardening results

from a certain disturbance* of the lattice. At least two factors determine the instantaneous work hardened state, namely, the number and the distribution of the dis-

distribution of slip bands suggests that the concept of equivalent work hardened states might fail.
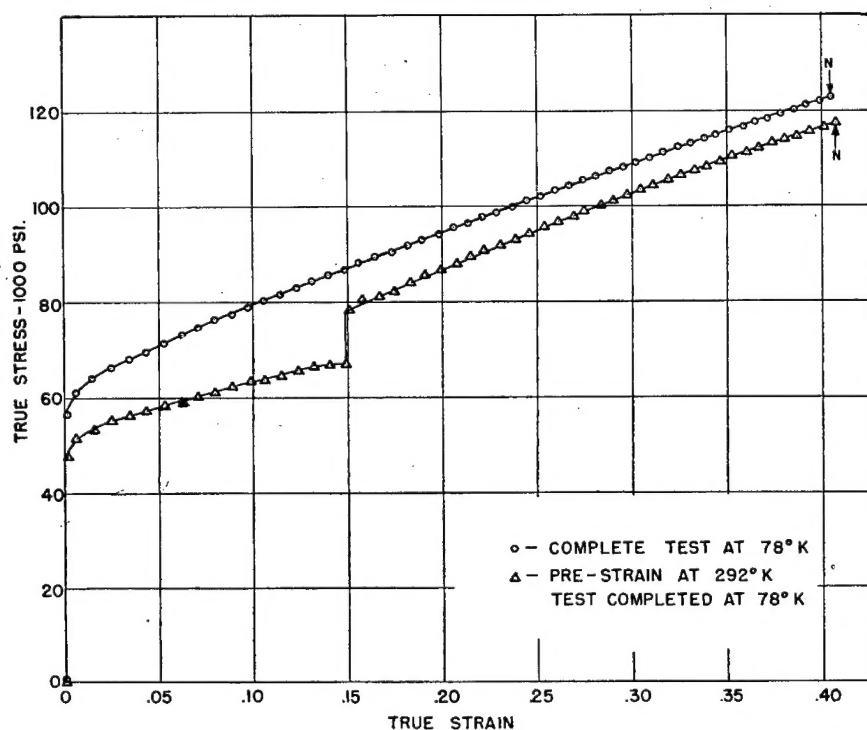
In an attempt to formulate a more sound

FIG 16—EFFECT OF PRE-STRAINING AT 292°K ON TRUE STRESS-TRUE STRAIN CURVE AT 78°K FOR 65-35 BRASS.

turbances. The same instantaneous flow stress, however, may then be obtained for several different sets of concentrations and distributions of disturbances. But the rate of strain hardening upon additional working at a fixed temperature and strain rate may differ for each set of concentrations and distribution of work hardening disturbances because the superposition of additional disturbances will not necessarily again yield new equivalent sets of work hardened states. The previously mentioned effect of temperature of strain rate on the

approach to a theory for creep of metals, a work hardened specimen of 2S-O was partially recovered at 400°F and its stress-strain curve was then determined and compared with the stress-strain curve of a virgin specimen.* Upon applying the method of coincidence, by shifting one stress-strain curve over the other, it was found that the two curves agreed at only one point and differed at all others. It is therefore apparent that the work hardened state, after partial recovery, is not equivalent to any work hardened state obtained by straining a virgin speci-

---

* The argument here is perfectly general and the reader may substitute dislocations, distortions or any other type of imperfection he believes promotes work hardening.

* J. E. Dorn and T. V. Cherian: unpublished research still in progress.
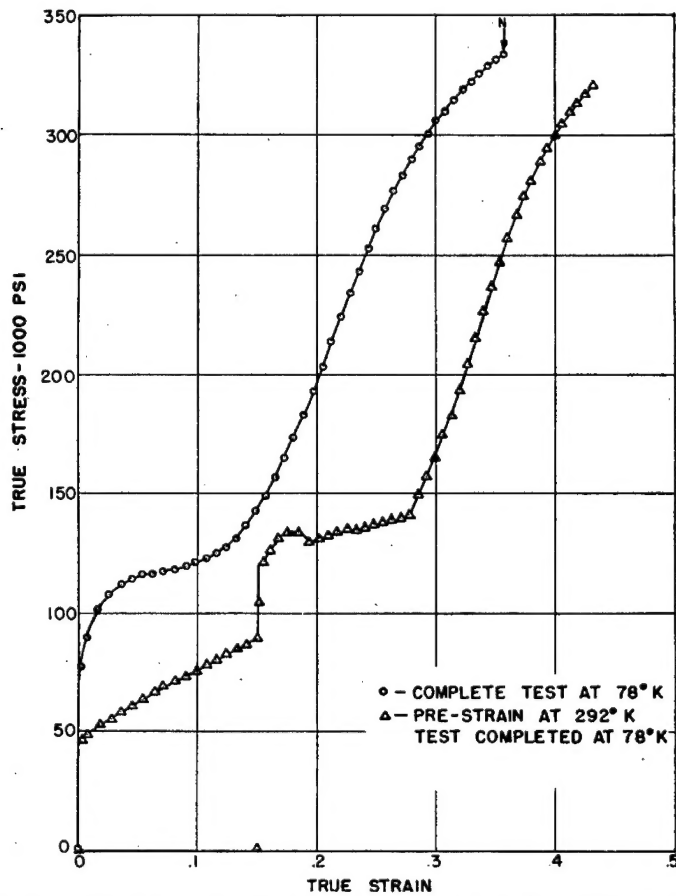
FIG 17—EFFECT OF PRE-STRAINING AT 292°K ON TRUE STRESS-TRUE STRAIN CURVE AT 78°K FOR 18-8 STAINLESS STEEL (TYPE 302).
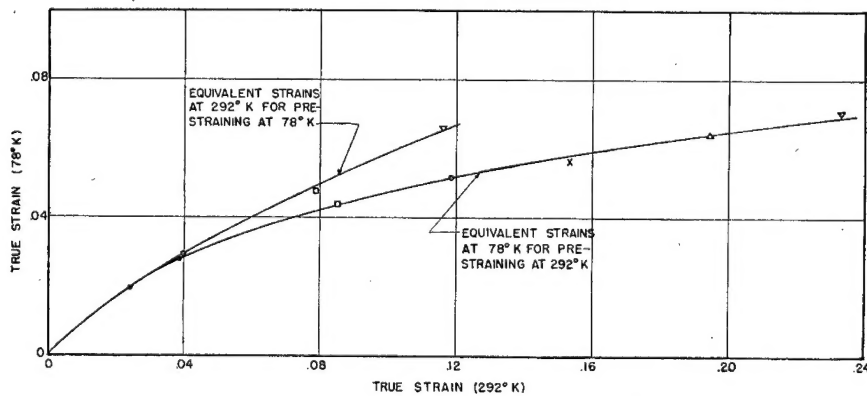

FIG 18—EQUIVALENT STRAINS AT LIQUID NITROGEN AND ATMOSPHERIC TEMPERATURES.

men. From these studies it is known that the work hardened state depends upon at least two microstructural changes that attend deformation.

compared it is evident that true equivalence is not obtained.

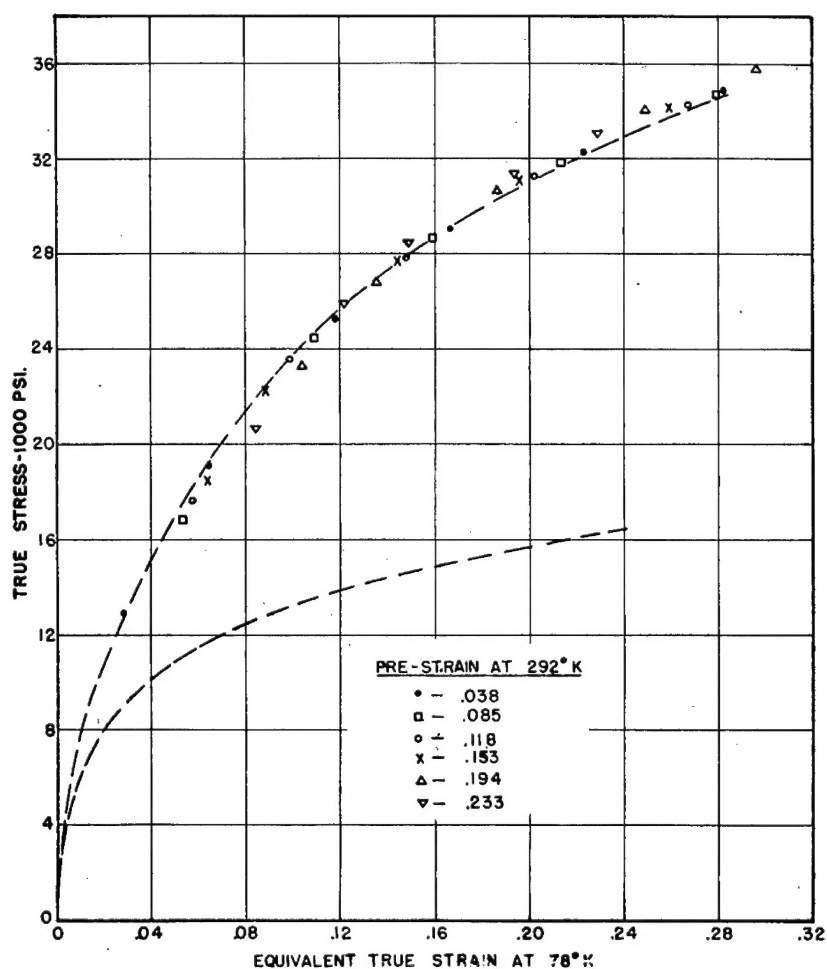Before substantial strides are possible from the phenomenological approach to

FIG 19—TRUE STRESS-TRUE STRAIN DATA PLOTTED IN TERMS OF EQUIVALENT TRUE STRAIN AT 78°K.

Although this observation shows that in general work hardened states cannot be identified by equivalent strains, it is yet possible that, in the absence of recovery, equivalent work hardened states might be found. The data given in Fig 19, when taken alone, appear to uphold this thought, but when the two curves of Fig 18 are

plasticity of metal, a clearer picture of the structural changes accompanying work hardening, recovery and relaxation will be needed.

CONCLUSIONS

1. The flow stress for plastic deformation of metals is not a simple function of

the instantaneous values of the strain, strain rate and temperature; it is sensitive to the entire thermal-mechanical history.

2. At lower test temperatures higher work hardened states are obtained for the same total strains; this factor is probably responsible for the well known lowering of the recrystallization temperature as the metal is cold worked at lower temperatures.

3. Simple history-modified strains did not yield equivalent work hardened states.

4. The failure of the mechanical equation of state is perhaps partly due to relaxation phenomena of low activation energy which can proceed at low temperatures and high strain rates.

5. Future progress in determining the plastic properties of metals will be largely dependent upon better understandings of the structural changes accompanying cold work and their effect on the flow stress.

ACKNOWLEDGMENTS

REFERENCES

1. P. Ludwick: Elements der Technologischen Mechanik. Julius Springer (1909), Leipsig.
2. C. Zener and J. H. Hollomon: Problems in Non-Elastic Deformation. *Jnl.* Appl. Phys., Feb. (1946).
3. J. H. Hollomon: Tensile Deformation. *Trans.* AIME (1945) **162**, 268.
4. H. Carpenter and J. M. Robertson: Metals. Oxford Press (1939) London.
5. C. W. MacGregor and J. C. Fisher: Tension Tests at Constant True Strain Rates. *Jnl.* Appl. Mech., Dec. (1945).
6. J. H. Hollomon and L. D. Joffee: Ferrous Metallurgical Design. John Wiley and Sons (1946).
7. S. Dushman, L. W. Dunbar and H. Hothsteiner: Creep of Metals. *Jnl.* Appl. Phys. (1944) **15**, 108.
8. J. H. Hollomon: The Mechanical Equation of State. *Met. Tech.* (Sept. 1946) TP 2304; *Trans.* AIME (1947) **171**, 535.
9. Kauzmann: Flow of Solid Metals from the Standpoint of the Chemical-rate Theory, *Trans.* AIME (1941) **143**, 57.
10. J. H. Hollomon and J. D. Lubahn: Plastic Flow of Metals. The Phys. Rev. (1946) **70** (1946) 775.
11. C. W. MacGregor and J. C. Fisher: Tension Tests at Constant True Strain Rates. *Jnl.* of Appl. Mech. *Trans.* A.S.M.E. (1945) **67**, A-217.
12. C. W. MacGregor and J. C. Fisher: *Trans.* Amer. Soc. Mech. Engr. (1946) **68**, AH–A16.
13. J. D. Lubahn: Derivation of Stress, Strain, Temperature, Strain-Rate Relations for Plastic Deformation. *Jnl.* Appl. Mech. **14**, No. 3, A 299–A 230.
14. J. H. Hollomon and J. D. Lubahn: The Flow of Metals at Elevated Temperatures. *Gen. Elec. Rev.* (1947) **50**, No. 2, 28–32: No. 4, 44–50.
15. G. I. Taylor: The Mechanism of Plastic Deformation of Crystals. *Proc.* Royal Soc. (1934) A-145, 362.
16. A. S. Nowick and E. S. Machlin: Dislocation Theory as Applied by N.A.C.A. to Creep of Metals. *Jnl.* Appl. Phys. (1947). **18**, No. 1, 79–87.
17. C. Zener and J. H. Hollomon: Effect of Strain Rate on the Plastic Flow of Steel. *Jnl.* Appl. Phys. (1944) **15**, 22–32.
18. F. Seitz and T. A. Read: Theory of the Plastic Properties of Solids. *Jnl.* Appl. Phys. (1941) **12**, 107.
19. J. C. R. Calvet: Acad. Sci. Paris (1935) **200**, 66.
20. A. W. McReynolds: Yield Stress Discontinuities and Plastic Flow in Aluminum. Sixth Quarterly Report to ONR, Univ. of Chicago, Aug. 1, 1947.
21. C. Barrett: The Structure of Metals. McGraw-Hill Co., (1943) N. Y.